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APPLICATIONS OF WAVELET TRANSFORM IN SIGNAL RECOGNITION AND DE-NOISING

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Abstract

The main aim of this paper is to show applications of a wavelet transform for an automatic signal classification and de-noising. We show that the wavelet decomposition of signal can be used for two applications. The first one, de-noising, allows to separate noise from the signal and to remove it before executing the recognition algorithm. The second approach, called multistage recognition, decomposes signal in wavelet bases, preparing it to a sequential recognition in many stages. We describe the characteristics of the presented methods and we discuss shortly their advantages and disadvantages.

1. INTRODUCTION

We consider a problem of signal classification to one of m classes. It means we assume the existence of m different generic patterns $f_1(t)$, $f_2(t), \ldots, f_m(t)$, for each class. In a fixed class with a pattern $f(t)$, the form of a signal disturbed by Gaussian noise is as follows

$$s(t_i) = f(t_i) + \sigma Z_i, \tag{1.1}$$

where $t_i = \frac{i}{p_0}$, for $i = 0, 1, \ldots, p_0 - 1$ are time samples and $\{Z_i\}$ are independent and identically distributed Gaussian random variables, $Z_i \sim \mathcal{N}(0; 1)$. The example of generic patterns and noisy signals is shown in Figure 1.

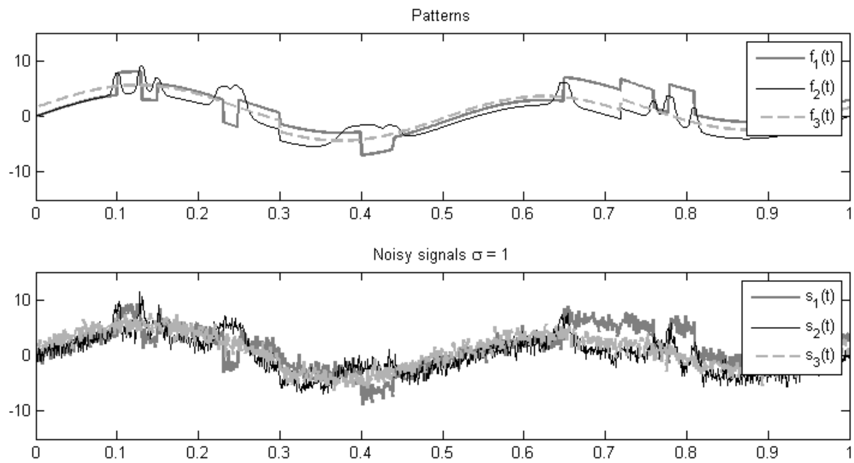

Figure 1: Generic patterns and noisy signals. Three classes of signals.

2. WAVELET TRANSFORM

2.1. Multiresolution signal representation

The multiresolution analysis of the space $L^2(\mathbb{R})$ is a sequence of approximation subspaces V_m, fulfilling the inclusion condition $V_m \subset V_{m+1} \subset \cdots \subset L^2(\mathbb{R})$. Every approximation space V_{m+1} can be presented as a simple sum of an approximation space V_m and a detail space W_m for smaller scale m, what gives a notation

$$V_K = V_M \bigoplus_{m=M}^{K-1} W_m. \tag{2.1}$$

We assume that $\phi(t)$ is a *scaling function* and $\psi(t)$ is a proper *mother wavelet*. Let $\phi_{mn}(t) = 2^{m/2}\phi(2^m t - n)$ be the basic function of approximation space V_m and $\psi_{mn}(t) = 2^{m/2}\psi(2^m t - n)$ be the basic function of detail space W_m for scale m [3].The signal approximation $s(t; K)$ for scale K has the form

$$s(t) \approx s(t; K) = \sum_n \alpha_{Mn}\phi_{Mn}(t) + \sum_{m=M}^{K-1}\sum_n \beta_{mn}\psi_{mn}(t) \tag{2.2}$$

where wavelet coefficients α_{Mn} and β_{mn} are given by the formulas $\alpha_{Mn} = \int_{\mathbb{R}} s(t)\,\phi_{Mn}(t)\,dt$ and $\beta_{mn} = \int_{\mathbb{R}} s(t)\,\psi_{mn}(t)\,dt$. A number n is a translation in time of wavelet functions.

Noisy signal (1.1) transformed by wavelet filtration to time-frequency domain is represented by the sequence of wavelet coefficients

$$W(s(t)) = (\alpha_M, \beta_M, \beta_{M+1}, \ldots, \beta_{K-1}) = \underline{x}, \tag{2.3}$$

where $\alpha_M = (\alpha_{Mn})_n$ - sequence of wavelet approximation coefficients for coarse scale M, $\beta_m = (\beta_{mn})_n$ - sequence of wavelet detail coefficients for scale $m = M, M+1, \ldots, K-1$.

2.2. Mallat's algorithm

Described in previous section, a decomposition (2.1) of an approximation space V_K was used by Mallat [7] in an algorithm construction. The algorithm calculates the coefficients of signal approximation $s(t;\, m)$ from space V_m in bases of subspaces V_{m-1} and W_{m-1}. The Mallat's algorithm recursively decomposes signal $s(t;\, m) \in V_m$ for $m = K, K-1, \ldots, M+1$ to approximation and detail components with the low-pass and high-pass filters, respectively. Basing on approximation coefficients $\{\alpha_{mn}\}$, we compute coefficients $\{\alpha_{m-1,n}\}$ and $\{\beta_{m-1,n}\}$ in the following way

$$\alpha_{m-1,n} = \sum_t h_t \alpha_{m,t+2n} \quad \text{and} \quad \beta_{m-1,n} = \sum_t g_t \alpha_{m,t+2n}, \tag{2.4}$$

where $\{h_t\}$ is a low-pass filter, and $\{g_t\} = \{(-1)^t h_{-t+1}\}$ – a complementary high-pass filter. The exemplary low-pass filters [3] are

- *Haar filter:* $h_0 = \frac{1}{\sqrt{2}}$, $h_1 = \frac{1}{\sqrt{2}}$,
- *Daubechies of order 2 filter:* $h_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}$, $h_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}$, $h_2 = \frac{3-\sqrt{3}}{4\sqrt{2}}$, $h_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}$.

As a result of the filtration, the approximation and the detail coefficients for the less accurate scale $m-1$ are down-sampled.

The block diagram of wavelet decomposition proposed by Mallat [7] is in Figure 2.

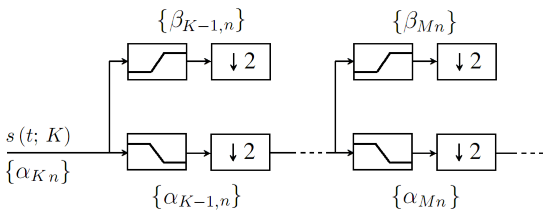

Figure 2: Block diagram of wavelet decomposition - Mallat's algorithm. Source: [5].

3. WAVELET DE-NOISING

The decomposition of signal in wavelet bases was used by Donoho and Johnstone for de-noising purposes. They proposed [1, 2] the *soft thresholding* of wavelet coefficients x of the vector $\underline{x}$ (see (2.2) and (2.3))

$$T_s(x,\,\lambda) = \begin{cases} (|x| - \lambda), & x \geq \lambda \\ 0, & |x| < \lambda \\ -(|x| - \lambda), & x \leq -\lambda \end{cases} \tag{3.1}$$

with the so called *universal threshold,*

$$\lambda = \widehat{\sigma}\sqrt{2log(p_0)}, \tag{3.2}$$

where $\widehat{\sigma} = MAD/0.6745$. MAD is median absolute value of normalized wavelet coefficients. The threshold λ is independent from wavelet decomposition level and this is the reason why it got the *universal* name. Donoho and Johnstone proved asymptotic optimality of *VisuShrink* estimator $\hat{f} = (W^{-1} \circ T_s \circ W)(s_i)$ in mean square error sense. The estimator $\hat{f}$ is reconstruction of signal f from noisy data $s_i = f(t_i) + \sigma Z_i$. It was shown that the *VisuShrink* [2] reconstruction $\hat{f}$ reduces noise and improves visual quality of images.

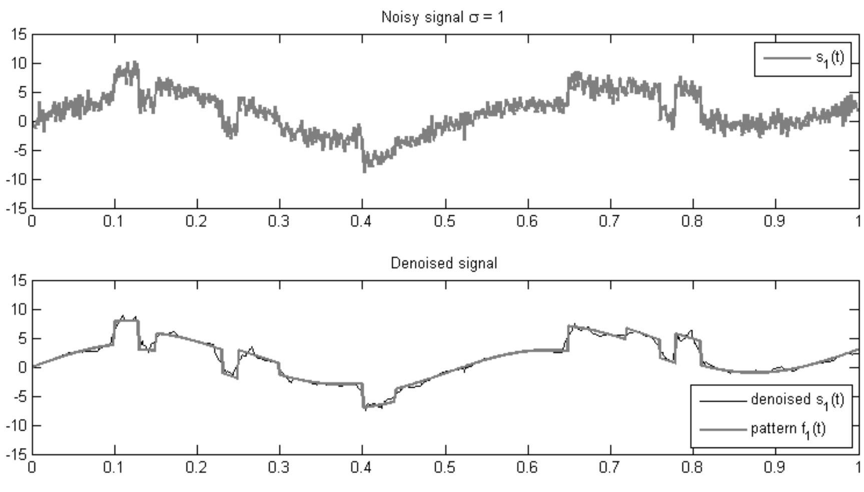

Figure 3: Noisy and de-noised signal. Wavelet decomposition performed with the use of 'db2' Daubechies wavelets of order 2.

4. MULTISTAGE SIGNAL RECOGNITION

The second application of wavelet decomposition of signal is its recognition in a sequential mode. It means we use only one sequence of wavelet coefficients on every stage. The wavelet sequences: $\alpha_M, \beta_M, \beta_{M+1}, \ldots, \beta_{K-1}$, (see (2.3)) represent an analyzed signal unambiguously, as in the formula:

$$s(t;K) = \underbrace{\sum_{n_1 \in \mathbb{Z}} \alpha_{Mn_1} \phi_{Mn_1}(t)}_{\in V_M} + \underbrace{\sum_{n_2 \in \mathbb{Z}} \beta_{Mn_2} \psi_{Mn_2}(t)}_{\in W_M} + \underbrace{\sum_{n_3 \in \mathbb{Z}} \beta_{M+1,n_3} \psi_{M+1,n_3}(t)}_{\in W_{M+1}} + \cdots + \underbrace{\sum_{n_N \in \mathbb{Z}} \beta_{K-1,n_N} \psi_{K-1,n_N}(t)}_{\in W_{K-1}}. \quad (4.1)$$

The main reason of the usage of the mentioned wavelet coefficient sequences from the decomposed representation is the lower number of coefficients in each sequence: $\alpha_M, \beta_M, \beta_{M+1}, \ldots, \beta_{K-1}$ then in the full representation by α_K. For example, the length of α_K for Haar transform is 2^{K-M}-times longer than the length of α_M. The wavelet decomposition causes a natural selection of signal coefficients taken into consideration by a classification algorithm. If the coarse representation by α_M is not good enough (in a risk value sense) to make the final decision of classification of the signal to a class, the signal will be assigned to a *macro-class* (a set of similar classes). On the next stage, the next sequence, i.e. β_M, is considered and its purpose is to precise the classification result, what is shown in the Figure 4. The procedure can be continued as long as we dispose with the sequences of wavelet detail coefficients, i.e. according to the form (4.1) on the last stage classifier chooses a final class on the basis of β_{K-1}.

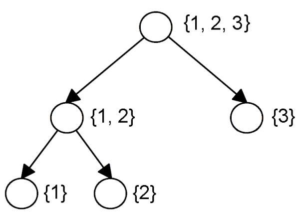

Figure 4: Decision tree with the transitional macro-classes (nodes: {1,2,3} and {1,2}) and the final classes (leaves: {1}, {2} and {3}).

The *multistage recognition* is a general schema and can be performed with every one-stage classifier (e.g. $k-NN$), as a building component of this complex procedure.

5. CONCLUSIONS

Every signal $s(t)$ may be represented by a sequence of wavelet coefficients. There are two processing we could exploit thanks to the representation of signal in wavelet bases: 1) the signal de-noising and 2) the multistage recognition of signal.

The *VisuShrink* estimator $\hat{f}$ of de-noised signal $s(t)$ has very good visual properties, but it leaves more non-zero coefficients than is necessary to execute recognition. There are other methods, as proposed by Johnstone and Silverman [4] thresholding *WaveletShrink* with threshold $\lambda_m = \sigma_m \sqrt{2log(p_0)}$ dependent on wavelet decomposition level m. But the future work is to find the threshold dedicated to classification problems, that selects proper coefficients, so the misclassification risk is minimized.

The multistage recognition of signal is a relatively fresh idea. An exemplary multistage classification of ECG signals can be found in [6]. It makes the complex problem of signal recognition easier, because of the reduction of coefficient vector dimension. But it only postpones the usage of signal details to the following stages. The only difficulty of the multistage approach is the opposite order of the coefficient sequences usage to the order they are produced by Mallat's algorithm. This forces the earlier preparation of signal decomposition before classification.

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